White Paper

Phase biases for ambiguity resolution: from an undifferenced to an uncombined formulation

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ABSTRACT

Integer ambiguity fixing has been recently applied to undifferenced GPS phase measurements to achieve precise positioning or PPP-AR. This technique relies on the estimation of specific phase biases combination identification and application. It works for dual-frequency GPS receivers. However, this phase biases representation is not suitable for triple frequency receivers, as the number of possible combinations for the biases can be dramatically high.

This paper presents a new uncombined formulation for phase biases, as proposed in the RTCM SSR standardization framework. The validity of such a representation for triple frequency receivers is demonstrated. In order to validate the proposed model, a technique to compute such phase biases is presented. Using real measurements, we show that the integer ambiguity nature of phase measurement is valid under the new uncombined formulation at least for all the ionosphere free combinations whose noise is small enough.

1) INTRODUCTION

Integer ambiguity fixing is now routinely applied to undifferenced GPS phase measurements to achieve precise positioning. Some implementations are also available in real-time [5]. This implementation allows performing PPP with ambiguity resolution at the cm level.

With the new modernized satellites capabilities, performing PPP with triple frequency measurements will be possible and therefore, the current dual-frequency formulation will not be applicable. There is also a need for a generalized formulation of phase biases for RTCM. In this RTCM framework, the definition of a standard is important to allow interoperability between the two components of a positioning system, i.e. the network side and the user one.

2) CLASSICAL UNDIFFERENCED FORMULATION

In this chapter, the formulation as defined in [1] is presented.

The notations are:

$$\gamma = \frac{f_1^2}{f_2^2}, \quad \lambda_1 = \frac{c}{f_1}, \quad \lambda_2 = \frac{c}{f_2}$$

where f_1 and f_2 are the two frequencies of the GPS system and c is the speed of light. For GPS L1 and L2 bands, $f_1 = 154 f_0$ and $f_2 = 120 f_0$, where $f_0 = 10.23\,\mathrm{MHz}$.

Pseudorange or code measurements, P_1 and P_2, are expressed in meters, while phase measurements, L_1 and L_2, are expressed in cycles.

The pseudorange and phase measurements are modeled as:

$$\begin{array}{rlllll} P_1 = & D_1 & +\Delta h_p & +\left(e+\Delta\tau_p\right) & & \\ P_2 = & D_2 & +\Delta h_p & +\gamma\left(e+\Delta\tau_p\right) & & \\ \lambda_1 L_1 = & D_1+\lambda_1 W & +\Delta h & -\left(e+\Delta\tau\right) & -\lambda_1 N_1 \\ \lambda_2 L_2 = & D_2+\lambda_2 W & +\Delta h & -\gamma\left(e+\Delta\tau\right) & -\lambda_2 N_2 \end{array} \tag{1}$$

where:

- D_1 and D_2 are the geometrical propagation distances between the emitter and receiver phase centers at f_1 and f_2 including troposphere elongation, relativistic effects, etc.
- W is the contribution of the wind-up effect (in cycles).
- e is the ionosphere elongation in meters at f_1. This elongation varies with the inverse of the square of the frequency and with opposite signs between phase and code.
- $\Delta h = h_i - h^j$ is the difference between receiver i and emitter j ionosphere-free phase clocks. Δh_p is the corresponding term for pseudorange clocks.
- $\Delta\tau = \tau_i - \tau^j$ is the difference between receiver i and emitter j offsets between the phase clocks at f_1 and the ionosphere-free phase clocks. By construction, the corresponding quantity at f_2 is $\gamma\,\Delta\tau$. Similarly, the corresponding quantity for pseudorange is $\Delta\tau_p$ (Time Group Delay).
- N_1 and N_2 are the two carrier phase ambiguities. By definition, these ambiguities are integers. Unambiguous phases measurements are therefore $L_1 + N_1$ and $L_2 + N_2$.

These equations (1) take into account all the biases related to delays and clocks. The four independent parameters $\Delta h, \Delta\tau, \Delta h_p, \Delta\tau_p$ are equivalent to the definition of one clock per observable. However, our choice of parameters emphasizes the specific nature of the problem by identifying reference clocks for pseudorange and phase (Δh_p and Δh) and the corresponding hardware offsets ($\Delta\tau_p$ and $\Delta\tau$). These offsets are assumed to vary slowly with time, with limited amplitudes.

According to [1], the measured widelane $\widetilde{N}_w$ (also called the Melbourne-Wübbena widelane) can be written as:

$$\left\langle \widetilde{N}_w \right\rangle = N_w + \mu_i - \mu^j \qquad (2)$$

where N_w is the integer widelane ambiguity, μ^j is the constant widelane delay for satellite j, μ_i is the widelane delay for receiver i (fairly stable for good geodetic receivers). The symbol $\langle\ \rangle$ means that all quantities have been averaged over a pass.

Integer widelane ambiguities N_w are then easily identified from averaged measured widelanes corrected for satellite widelane delays. Once integer widelane ambiguities N_w are known, the ionosphere-free phase combination can be expressed as

$$Q_c = D_c + \lambda_c W + h_i - h^j - \lambda_c N_1 \qquad (3)$$

where $Q_c = \dfrac{\gamma\lambda_l L_1 - \lambda_2\left(L_2 + N_w\right)}{\gamma - 1}$ is the ionosphere-free phase combination computed using the known N_w ambiguity, D_c is the propagation distance, h_i is the receiver clock, h^j is the satellite clock. N_1 is the remaining ambiguity associated to the ionosphere-free wavelength λ_c (10.7 cm).

The complete problem is thus transformed into a single frequency problem with wavelength λ_c and without any ionosphere contribution.

Many algorithms can be used to solve equation (3) over a network of stations. If D_c is known with sufficient accuracy (typically a few centimeters, which can be achieved using a good floating ambiguity solution), it is possible to simultaneously solve for N_1, h_i and h^j.

The properties of such a solution have been studied in details. A very interesting property of the h^j satellites clocks is, in particular, the capability to directly fix the N_1 values of a receiver which has not been part of the initial network [1].

The majority of PPP-AR implantations are based on the identification and use of the two quantities μ^j and h^j. These quantities may be called wilelane biases and integer phase clocks [1], decoupled clock model [2], or Uncalibrated Phase Delays [3], but are of the same nature.

3) A REAL-TIME PPP-AR IMPLEMENTATION, THE PPP-WIZARD DEMONSTRATOR

This PPP-AR technique has been successfully implemented by CNES in real-time in the PPP-WIZARD demonstrator since 2010 [5]. In this demonstrator and in the framework of the IGS RTS [6] and RTCM [7] the GPS and Glonass constellation orbits and clocks are computed. Additional biases for GPS ambiguity resolution are computed and broadcast to the user. The demonstrator also provides an open-source implementation of the method on the user side, for test purposes. Centimeter positioning accuracy in real time is obtained on a routine basis.

4) LIMITATIONS OF THE BIASES FORMULATION

The current formulation works but it has several drawbacks:

- The chosen representation is dependent on the implemented method. Even if the nature of biases is the same, their representation may be different according to the underlying methods and make it difficult for a standardization of the biases messages.
- The user side must implement the same method as the one used in the network side. Otherwise, the user side would have to convert the quantities from one method to another, leading to potential bugs or misinterpretations.
- It is limited to the dual-frequency case. There are only two quantities to be computed in the dual-frequency case (${\mu_{12}}^j$ and ${h_{12}}^j$), but in the triple frequency case, there is much more possible combinations. For example, one can have (this is a non-exhaustive list): ${\mu_{12}}^j$ ${\mu_{15}}^j$ ${\mu_{25}}^j$ ${h_{12}}^j$ ${h_{15}}^j$ ${h_{25}}^j$ and other ionosphere-free combinations like phase widelane-only or even phase ionosphere-free and geometry-free combination.

5) NEW MODEL AS DEFINED IN THE RTCM SSR

The new model, as proposed by the RTCM SSR group for phase biases messages is based on the idea that the phase bias is inherent to each frequency. Thus, instead of making specific combinations, one phase bias per phase observable is identified and broadcast [8].

It is noted that this convention was adopted a long time ago for code biases. Indeed, in the RTCM framework, and unlike the standard DCB convention where code biases are undifferenced but combined, the RTCM SSR code biases are defined as undifferenced AND uncombined [9].

The general model for uncombined code and phase biases is therefore:

$$\begin{aligned}
P_1' &= P_1 + \Delta b_{P_1} = D + e + \Delta h_p \\
P_2' &= P_2 + \Delta b_{P_2} = D + \gamma e + \Delta h_p \\
\lambda_1 L_1' &= \lambda_1 \left(L_1 + \Delta b_{L_1}\right) = D - e + \Delta h_p - \lambda_1 N_1 \\
\lambda_2 L_2' &= \lambda_2 \left(L_2 + \Delta b_{L_2}\right) = D - \gamma e + \Delta h_p - \lambda_2 N_2
\end{aligned} \tag{4}$$

Time Group Delays τ and phase clocks h in equation (1) are replaced by code and phase biases (Δb_P and Δb_L respectively). RTCM code and phase biases correspond to the satellite part of these biases.

The ‘ notation denotes the ‘unbiasing’ process of the measurements.

Here, the clock definition is crucial. As the biases are uncombined, there are referenced to the clocks. The convention chosen for the standard is natural: it is the same as the IGS one, i.e. Δh_p in our notations.

This new model can be extended to the triple frequency case very easily (5), as it does not involve explicit dual frequency combinations:

$$\begin{aligned}
P_1' &= P_1 + \Delta b_{P_1} = D + e + \Delta h_p \\
P_2' &= P_2 + \Delta b_{P_2} = D + \gamma_2 e + \Delta h_p \\
C_5' &= C_5 + \Delta b_{C_5} = D + \gamma_5 e + \Delta h_p \\
\lambda_1 L_1' &= \lambda_1 \left(L_1 + \Delta b_{L_1}\right) = D - e + \Delta h_p - \lambda_1 N_1 \\
\lambda_2 L_2' &= \lambda_2 \left(L_2 + \Delta b_{L_2}\right) = D - \gamma_2 e + \Delta h_p - \lambda_2 N_2 \\
\lambda_5 L_5' &= \lambda_5 \left(L_5 + \Delta b_{L_{52}}\right) = D - \gamma_5 e + \Delta h_p - \lambda_5 N_5
\end{aligned} \tag{5}$$

This new model simplifies the concept of phase biases for ambiguity resolution. This representation is very attractive because no assumption is made on the method used to identify phase biases on the network side. All the implementations are valid if they respect this proposed model. It also allows convenient interoperability if the network and user side implement different ambiguity resolution methods.

The following table summarizes the different messages used for PPP-AR in the context of RTCM SSR:

Parameter Nature	RTCM SSR message	Quantity
GPS/Glonass orbits/clocks	1066/1066	D, h_p
GPS code biases	1059/1065	b_P
GPS phase biases	1265	b_L

Table 1: RTCM SSR messages for PPP-AR

6) UNCOMBINED PHASE BIASES ESTIMATION IN THE DUAL FREQUENCY CASE

The new phase biases identification in the two frequency case is straightforward. There are two biases (b_{L_1}, b_{L_2}) to be estimated using two combinations (*μ* and *h*).

The problem to be solved is described in the next figure:

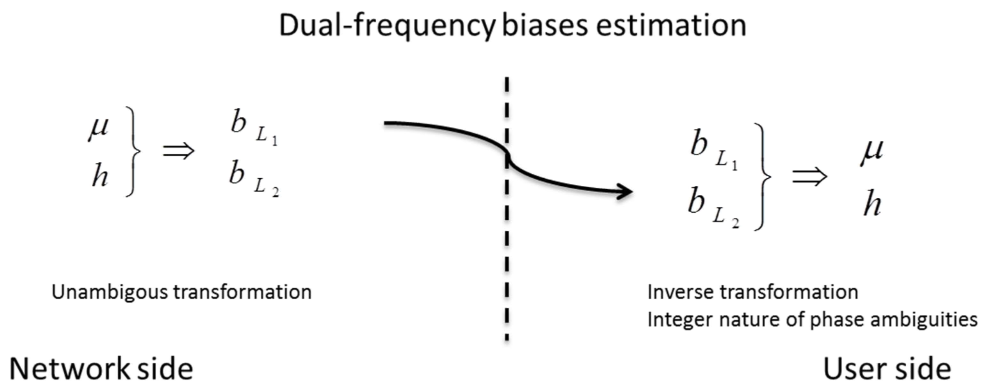

Figure 1: phase biases estimation in the dual frequency case

This problem can be solved very easily on the network side by means of a 2*2 matrix inversion:

$$\begin{pmatrix} b_{L_1} \\ b_{L_2} \end{pmatrix} = \frac{1}{\gamma_2\lambda_1 - \lambda_2} \begin{pmatrix} -\lambda_2 & 1 \\ -\gamma_2\lambda_1 & 1 \end{pmatrix} \begin{pmatrix} \mu - \alpha_{21} b_{P_1} - \alpha_{22} b_{P_2} \\ (\gamma_2 - 1)(h - h_P) \end{pmatrix} \text{ with } \begin{aligned} \alpha_{21} &= \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} / \lambda_1 \\ \alpha_{22} &= \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} / \lambda_2 \end{aligned}$$

<u>Note</u>: all the quantities denote the satellite part of the Δ operator defined above.

7) UNCOMBINED PHASE BIASES ESTIMATION IN THE TRIPLE FREQUENCY CASE

The triple-frequency biases identification is tricky due to the need, using only three biases, to keep the integer nature of phase ambiguities on all viable ionosphere free combinations, and in particular combinations that were not used in the identification process. At this level, one cannot make assumptions on what kind of combinations will be used by a user.

The problem to be solved is described in the following figure:

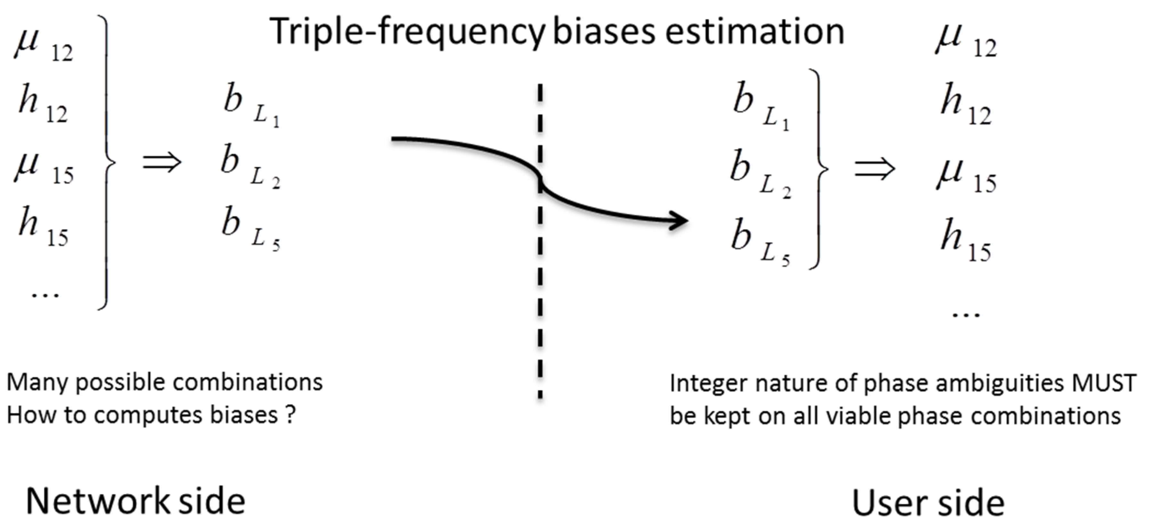

Figure 2: phase biases estimation in the triple frequency case

As an example, a naïve solution would be to identify the extra-widelane ${\mu_{25}}^{j}$ phase biases using the dual frequency widelane approach and then identify the b_{L_5} bias. Given the high wavelength of the extra-widelane combination, such identification would be very easy. However, the corresponding bias would be only helpful for extra-widelane ambiguity identification, and its noise would prevent its use for widelane 15 ambiguity resolution or other useful combinations available in the triple-frequency context.

Each independent phase bias can be directly estimated in a filter; however, in order to keep ascending compatibility with the dual frequency case during the deployment phase of the new modernized satellites, we have chosen to stay in the old framework, i.e. to work with combinations of biases. The resolution method is the following:

- The widelane biases, i.e. the identification of all the $b_{Li} - b_{Lj}$ quantities, are solved. For this computation and in order to have an accurate estimate of these biases, the two MW-widelanes biases μ_{12} , μ_{15} are used coupled to an additional phase bias, which is given by the triple-frequency ionosphere-free phase combination with the integer widelanes ambiguities already fixed. This last combination using only phase measurements is much more accurate than MW-widelanes. The system to be solved is redundant and the noise of the different equations has to be chosen carefully.

- The remaining bias (b_{L1}) is estimated using the traditional ionosphere free phase combination L_1,L_2.

This computation has been implemented in the CNES real-time analysis center software, and since 09/15/2014, CNES broadcasts on the CLK93 stream phase biases compatible with this triple-frequency concept.

8) REAL DATA ANALYSIS: THE UNCOMBINED PHASE BIAS CONCEPT AND THE INTEGER AMBIGUITY NATURE OF PHASE MEASUREMENTS

To prove the validity of the concept, we compute several ambiguities combinations using real data. The process is the following:

A) Look for good receiver locations having a high number of block IIF satellites in view for a period of time exceeding 30 minutes, and choose among them, one equipped with a MGEX receiver. The CPVG (Cape Verde) station from the REGINA network was chosen for the time span on the 28th of Sep 2014 between 19h and 20h UTC. Over this period 4 block IIF satellites were visible simultaneously (PRNs 1, 6, 9, 30), for a total of 14 GPS satellites in view.

B) Record a compatible phase biases stream. The CLK93 stream was recorded during the time span of the experiment.

C) Perform a PPP using the measurements, CLK93 corrections and biases to estimate the propagation distance, the troposphere delay, the receiver clock and phase ambiguities estimates according to equation (5).

D) For different ambiguity estimates, compute and plot the obtained residuals.

We present in the following graphs various ambiguity residuals, for the 4 block IIF satellites in view. The values of each ambiguity are offset by an integer value for clarity purposes.

Melbourne-Wübbena extra-widelane

The following figure represents the MW extra-widelane (between frequencies 2 and 5) ambiguity estimation using our process:

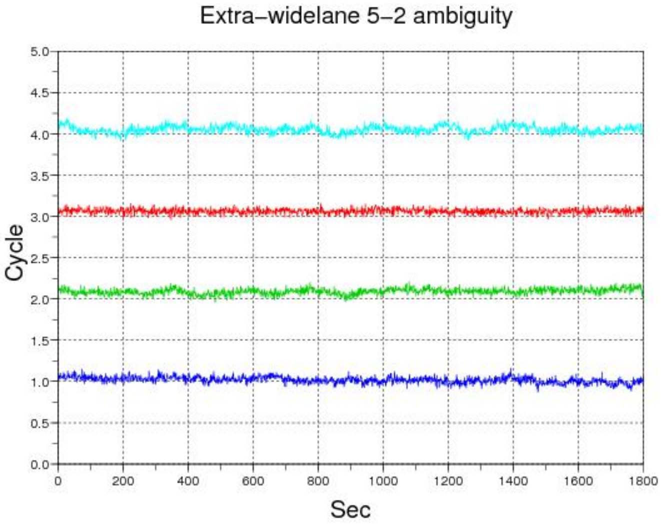

Figure 3: ambiguity residuals for the extra-widelane combination

The MW extra-widelane ambiguity has a wavelength of 5.86 m. The noise of the combination expressed in cycle is very low and the integer nature of ambiguities on this combination is clearly visible.

Melbourne-Wübbena widelanes

The following figure represents the MW-widelanes (regular 1-2 and 1-5).

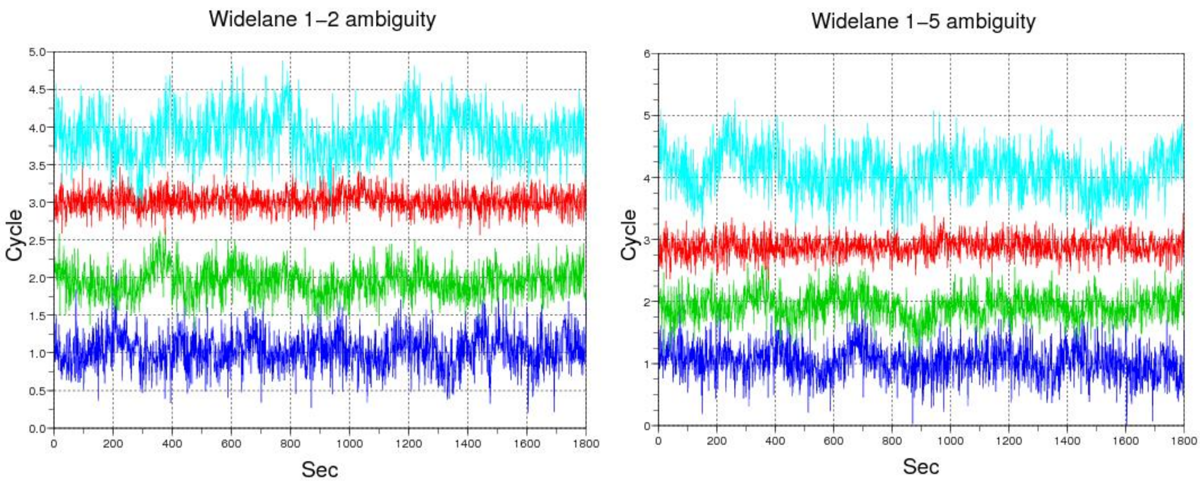

Figure 4: ambiguity residuals for widelanes combinations

Here again, the integer nature of the 4 ambiguities is clearly visible.

Widelanes-Only ionosphere-free phase

In the triple frequency context, there is a possibility to form an ionosphere-free combination of the 3 phase observables [4]. This combination has an important noise amplification factor (>20), but would allow to perform decimeter accuracy PPP using only the widelane integer ambiguities solved and if the corresponding phase biases are accurate. In addition, it can be shown that the wavelength of the widelane ambiguity when the extra-widelane ambiguity is solved is about 3.4 m. It means that the remaining widelane using this combination can be solved if the position is accurate enough (few tens of cm) and the extra-widelane is known. The following figure shows such a case, i.e. the residuals of the widelane ambiguity using this combination and assuming that the extra-widelane is already solved.

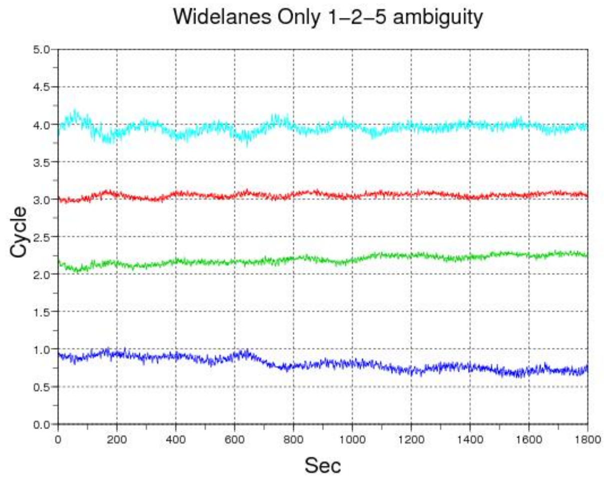

Figure 5: ambiguity residuals for widelane-only ionosphere free combinations

This is such a case where the solution is the most biased (the dark blue curve). This is mainly due to the difficulty to estimate the phase biases on this combination accurately using only a few number of block IIF satellites. We hope that in the future the increasing number of modernized satellites will help such biases estimation.

N1 ionosphere-free phase

The following figures shows the 3 possible ambiguities estimate using the ionosphere-free phase combination with 2 measurements (we assume that the corresponding widelane has already been solved). In each case, the computed biases allow to retrieve easily the integer nature of the N1 ambiguity.

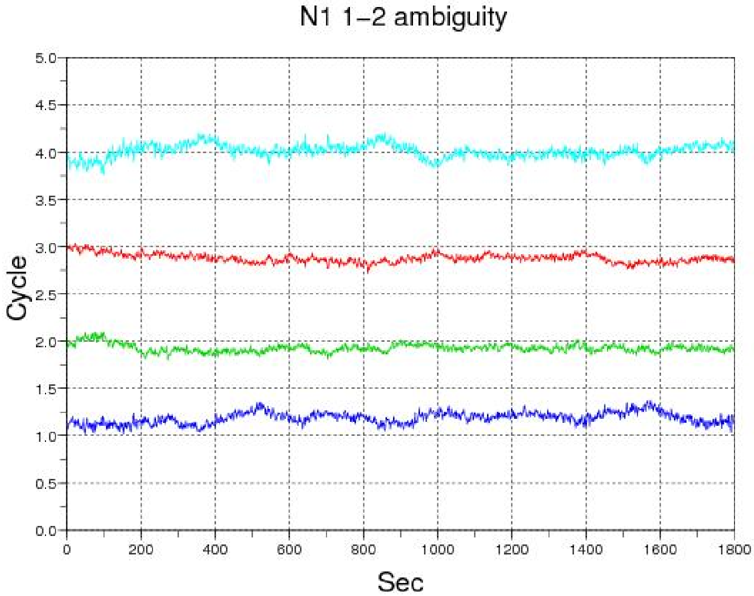

Figure 6: ambiguity residuals for the N1 combination using fixed 1-2 widelane

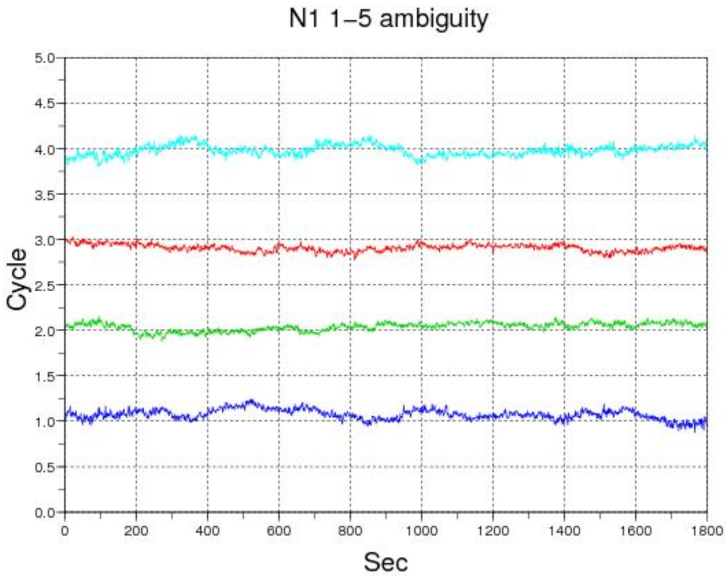

Figure 7: ambiguity residuals for the N1 combination using fixed 1-5 widelane

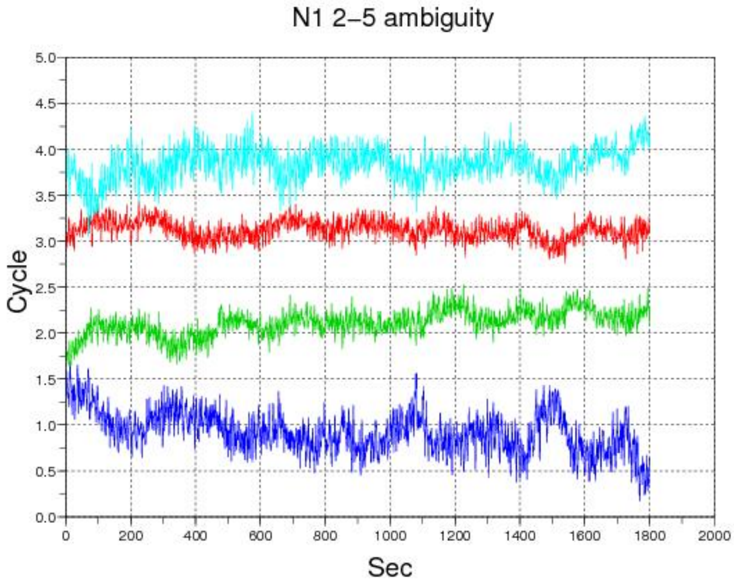

Figure 8: ambiguity residuals for the N1 combination using fixed 2-5 widelane

9) CONCLUSION

The new phase bias concept proposed by the RTCM SSR has been successfully implemented in the CNES IGS real-time analysis center. This new concept represents the phase biases in an uncombined form, unlike the previous formulations. It has the advantage of the unification of the different proposed methods for ambiguity resolution and prepares the future, for example in the triple-frequency case.

We have shown on real data that this concept is valid, i.e. the integer ambiguity nature of phase measurements is conserved for various useful observable combinations.

The impact of ambiguity convergence time in the triple-frequency context is left for further research.

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