Real Time Precise GPS Constellation Orbits and Clocks Estimation using Zero-difference Integer Ambiguity Fixing

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BIOGRAPHY

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ABSTRACT

A method to solve the GPS zero-difference measurement equations with integer ambiguities has been recently introduced [1]. When the method is applied to data from a global network of GPS receivers it provides a consistent set of satellite orbits and clocks, which have an 'integer' property: phase residuals for any receiver computed using these orbits and clocks easily reveal integer ambiguities.

The paper focuses on the application of this novel approach to the computation of real-time orbits and clocks for the GPS constellation. The benefit of using these products for real-time Precise Point Positioning (PPP) of user receivers (with integer ambiguity fixing) is demonstrated.

In this method real-time corrections to extrapolated IGS IGU orbits are estimated at the same time as all other relevant parameters by a Kalman which processes measurements from a world-wide network of 50 IGS stations. The filter performs zero-difference ambiguity fixing in real-time. One month of data was processed (July 2007) to estimate the achieved precision. Relative to IGS final orbits, the 3-D precision of the real-time orbits is about 3 cm RMS.

When these constellation orbits and clocks are used to perform real-time PPP for receivers outside of the reference network, the precision obtained using zero-difference integer ambiguity fixing is close to 2 cm RMS. This is about five times better than standard solutions which rely upon floating ambiguity fixing.

INTRODUCTION

Integer ambiguity resolution is routinely applied to double differenced GPS phase measurements to achieve precise positioning of ground receivers. Double differencing removes common errors between the different signal paths and minimizes the size of the problem by eliminating all clock contributions. However, it is possible to find the integer ambiguities on zero-difference phase measurements by processing these common biases, which can be expressed as simple functions of satellite/receiver internal delays.

This new method for zero-difference ambiguity fixing is presented in [1]. Applications to precise point positioning (PPP) are given in [1], to time transfer in [2], to real-time PPP in [3] and to orbit determination for space-borne receivers in [4]. The current paper extends the real-time applications of this ambiguity fixing technique. In particular it was shown in [3] that real-time zero-difference ambiguity fixed PPP is possible for a regional network, using limited precision orbits for the GPS satellites (such as IGS predicted orbits).

This paper extends this result to a global network. In this case, GPS satellite orbits need to be precise, and the constellation orbits have to be estimated in the Kalman filter as the same time as clocks and PPP parameters. Resulting real-time constellation orbits and clocks benefit from the increased observability provided by zero-difference ambiguity fixed phase measurements.

The zero-difference ambiguity fixed network solution involves two steps:

- zero difference real-time wide-lane ambiguity fixing at receiver level using known satellite delays. This wide-lane ambiguity fixing is performed independently on each receiver, knowing a limited set of satellite biases, leading to ionosphere-free phase expressions where only the ambiguity on the first frequency remains to be fixed (for GPS, the equivalent wavelength is 10.7 cm).

- zero difference real-time narrow-lane global ambiguity blocking over the station network and computation of the corresponding constellation orbits and clocks.

The state vector of the Kalman filter used in the second step is detailed in section 3. A real-time narrow-lane ambiguity fixing strategy is also proposed in section 3. This technique relies upon a mixed floating and integer filter, which fixes ambiguities to their integer values as soon as certain conditions are met, while maintaining consistency of all integer ambiguities over the entire network.

Section 4 details the tests which were conducted to assess the precision of this technique. Two orbit and clock solutions were computed using one month of data to compare the performance of a purely floating process with the ambiguity fixed solution. Section 5 presents the results of the comparison of these two solutions to IGS reference orbits and clocks (both IGS IGR and final products). It is shown that the real-time fixed ambiguities solution significantly outperforms its floating counterpart, with a precision close to that of time differed reference products.

These real-time constellation orbits and clocks are then used to evaluate the performance of user receiver Precise Point Positioning (PPP). Section 6 compares residuals obtained with time-differed reference solutions to the real-time values. Here again the real-time integer fixed solution is close to the reference. Real-time products are then input into a PPP filter to demonstrate that integer fixed real-time orbits and clocks combined with zero-difference integer fixed PPP leads to real-time positioning with 2 cm RMS precision.

To our knowledge, this is the first complete real-time solution for the GPS constellation orbits and clocks estimation combined with PPP using integer ambiguity fixing techniques. Standard real-time PPP solutions such as those of Omnistar [11] or StarFire [12], or the JPL [10] and NrCan [13] Automatic Precise Positioning Services all use a method based on zero-difference floating ambiguities.

IGS analysis centers [7] produce solutions with double-differences integer ambiguity fixing, but it is not easy to know exactly what specific strategy is employed on each of the products (Near real time, rapid or final) [8].

Other centers like NrCan are investigating issues related to zero-difference ambiguity fixing, with alternate methods. They plan to introduce the so-called "Decoupled Clock Model" [5, 6] in their future products [9].

2. GENERAL OVERVIEW OF THE METHOD

The detailed formulation is presented in [1, 3, 4]. Only the key characteristics are summarized hereafter.

According to [1], the measured widelane $\widetilde{N}_w$ (also called the Melbourne-Wübbena widelane) can be written as:

$$\left\langle \widetilde{N}_w \right\rangle = N_w + \mu_i - \mu^j \qquad (1)$$

where N_w is the integer widelane ambiguity, μ^j is the constant widelane delay for satellite j, μ_i is the widelane delay for receiver i (fairly stable for good geodetic receivers). The symbol $\langle\ \rangle$ means that all quantities have been averaged over a pass.

As a consequence, integer widelane ambiguities N_w can be easily identified after correcting averaged measured widelane for satellite widelane delays.

Once integer widelane ambiguities N_w are fixed, the ionosphere-free phase combination can be expressed as

$$Q_c = D_c + h_i - h^j - \lambda_c N_1 \qquad (2)$$

where $Q_c = \dfrac{\gamma\lambda_1 L_1 - \lambda_2 (L_2 + N_w)}{\gamma - 1}$ is the ionosphere-free phase combination computed using the known N_w ambiguity, D_c is the propagation distance, h_i is the receiver clock, h^j is the satellite clock. N_1 is the remaining ambiguity associated to the ionosphere-free wavelength λ_c (10.7 cm).

The complete problem is thus transformed into a single frequency problem with wavelength λ_c and without any ionosphere contribution.

Many algorithms can be used to solve the set of equations (2) over a network of stations (it is even possible to use standard double difference methods). If D_c is known with sufficient precision (typically a few centimeters, which can be achieved using a good floating ambiguity solution), it is possible to simultaneously solve for N_1, h_i and h^j [1, 4].

3. REAL TIME IMPLEMENTATION OF THE METHOD

3.1. REAL TIME WIDELANE AMBIGUITY FIXING

The zero-difference widelane integer ambiguity fixing uses equation (1). In real-time processing, the averaging is not performed on a pass by pass basis, but by a sliding window. The goal of this averaging is to reduce measurement noise well below one widelane cycle. The optimal window length is thus the result of a trade-off between the success rate of the ambiguity fixing and the delay introduced in the estimation process. Our experience shows that 30 minutes is a good window length for widelane fixing, for contemporary receivers. In our set-up, the widelane ambiguity fixing is performed by a pre-processing module.

3.2. REAL TIME N1 FIXING

The real-time N1 integer ambiguity fixing is performed directly by the main Kalman filter. This filter works in the floating domain for all its parameters except for N_1 ambiguities which are set to their integer value once they are known with enough confidence.

At the beginning of a pass, neither N_w nor N_1 are known, so the covariance of the ambiguity is set to an initial value (typically 10 m), and the filter works entirely in floating mode. After 30 minutes, the pre-processing module fixes N_w to its integer value. N_1 can be then fixed in the filter. At this stage, two different configurations may arise:

1) The pass is between a satellite and a station that are linked by other measurements whose ambiguities have already been fixed (fig 1). Because of the implicit closure of the equations, the covariance on N_1 is already close to the phase measurement noise (typically 1 cm). In this case, N_1 should already be close to an integer, but not yet fixed. It is then fixed to this integer value.

2) The new pass is not constrained by any other measurements with fixed ambiguities (this is detected by testing the covariance of the ambiguity parameter in the filter). N_1 can than be set to any integer.

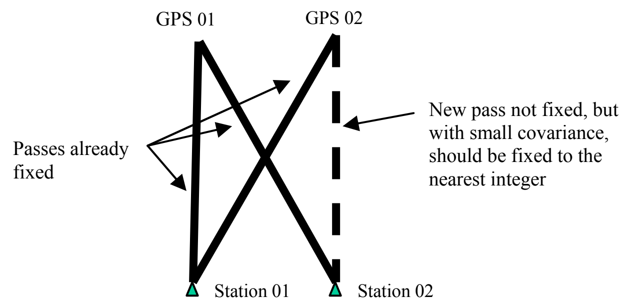

Fig. 1: connected ambiguities

The ambiguity fixing process is performed by adding a constraint equation to the filter. The fixing of a new ambiguity impacts the whole network; it can trigger by continuity the fixing of other ambiguities not yet fixed.

3.3. KALMAN FILTER EXTENSION TO A GLOBAL NETWORK

In [3] this technique was applied to a local network, using externally provided orbits for the GPS constellation (IGS predicted orbits). Over short distances, the GPS orbit error projected onto the satellite-station line-of-sight can be fairly well compensated by the adjusted satellite clock. However, over inter-continental distances this is no longer true, and GPS orbits have to be adjusted in the filter. In practice rather than starting from scratch, it is easier to start from a good predicted orbit (such as an IGS predicted orbit) and to solve for orbital corrections in the local orbital frame (radial, tangential or along-track, normal or across-track).

The parameters estimated in the extended filter are thus the following:

Table 1: Kalman filter parameters

Parameter nature	Quantity	Typical number
satellite phase clock	1 per satellite	34
station phase clock	1 per station	50
satellite code/phase bias	1 per satellite	34
station code/phase bias	1 per station	50
zenith troposphere delay	1 per station	50
station coordinates corrections	3 per station	50*3
satellite orbit corrections (R,T,N)	3 per satellite	34*3
Phase ambiguities	12 per station (max)	50*12
		1070

The filter is fed with ionosphere-free pseudo-range and phase measurements with measurement noises of 10 m and 1 cm respectively.

With a global network of around 50 stations there are about 500 phase measurements and 500 pseudo-range measurements at each time step. Because here we are primarily interested in the GPS orbit solutions (which have slow variations), the time step is set to 5 minutes.

The parameter model noises are set to the following values (for a 5 min sampling):

Table 2: Filter model noise

Parameter nature	Value	Comments
Phase satellite clock	∞	Purely stochastic
Phase station clock	∞	Purely stochastic
Code/Phase satellite bias	0	Set to 0
Code/Phase station bias	0	Set to 0
Zenith troposphere delay	1 mm	
Station coordinates (X, Y, Z ITRF)	0	Coordinates are set to 0
satellite orbit corrections(R, T, N)	(0, 4 mm, 2 mm)	Radial correction is set to 0
Phase ambiguities	0	Ambiguities are constant during a pass (initial covariance set to 100 m at the beginning of the pass)

Radial corrections to GPS orbits are highly correlated to clocks. For this reason these parameters are not estimated in the filter. As the radial error of the initial orbits is already below 2 cm, well below the narrow-lane wavelength (10.7 cm), not correcting radial orbit error does not create difficulties. Similarly as the primary focus of the test is to evaluate the precision with which satellites orbits and clocks can be estimated in real-time using the zero-difference ambiguity fixing technique, stations coordinates are not estimated in the filter; station coordinates are set to their ITRF 2005 values.

It should be noted that the orbit solution obtained with this method is purely geometric and not dynamical. Auxiliary parameters (such as Earth orientation parameters) are not estimated. In this sense, this solution is not optimal, however it will be shown that it has all the necessary properties to be used in a PPP solution.

4. TEST SET-UP

To evaluate the precision of the orbit and clock solutions obtained in real-time using the zero-difference ambiguity fixing filter, a statistical analysis was performed on a large data set. Stations measurements, initial orbits and reference orbits were downloaded from the IGS [14].

4.1. STATION NETWORK

A network of 50 IGS stations more or less evenly distributed around the globe was used in the tests. On occasion one station (here Reyk) was left out of the network to act as a test user receiver.

Figure 2 displays the number of stations seen by the GPS satellites as a function of the longitude and latitude of the sub-satellite point. The black spots correspond to satellites in eclipse on that day. A significant North-South asymmetry is clearly visible on this plot, however it should be noted that there are no areas where GPS satellites see less than 6 stations at a given time, which ensures the continuity of clock solutions.

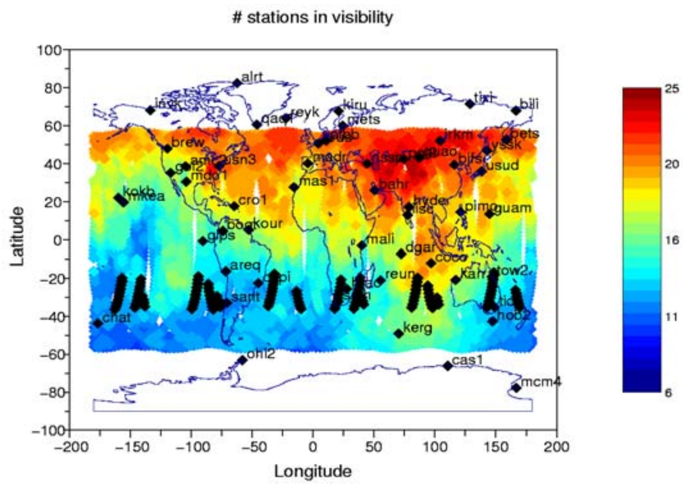

Fig. 2: Network of stations

4.2. INITIAL ORBITS

The filter estimates corrections to initial input GPS satellite orbits. In this test initial orbits are extrapolated IGS IGU orbits. These orbits cover a period of 2 days where the first day is the result of an orbit determination, and the second day is extrapolated. Given the delays introduced by the availability of these orbits, only the last 12 hours are effectively usable in the real-time filter. As new IGU solutions are produced by the IGS every 6 hours, this is not a limitation.

In the test each of the 4 IGU orbit files available per day are processed independently. The Kalman filter is initialized on the observed part of the IGU orbits, so the covariance is converged when the extrapolated section is reached, in order to simulate a steady-state mode of operation (cf. Figure 3).

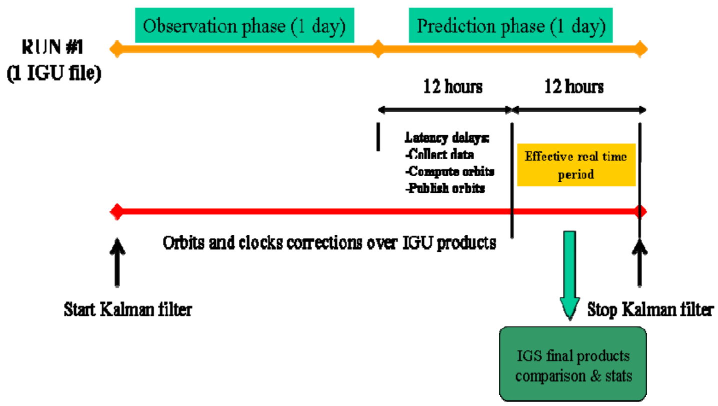

Fig. 3: processing set-up

120 IGU orbit files were processed in the test, covering the month of July 2007. This leads to 120 independent runs. Real-time filter performance is only evaluated over the last 12 hours of each run.

4.3. PHASE RESIDUALS PER STATION

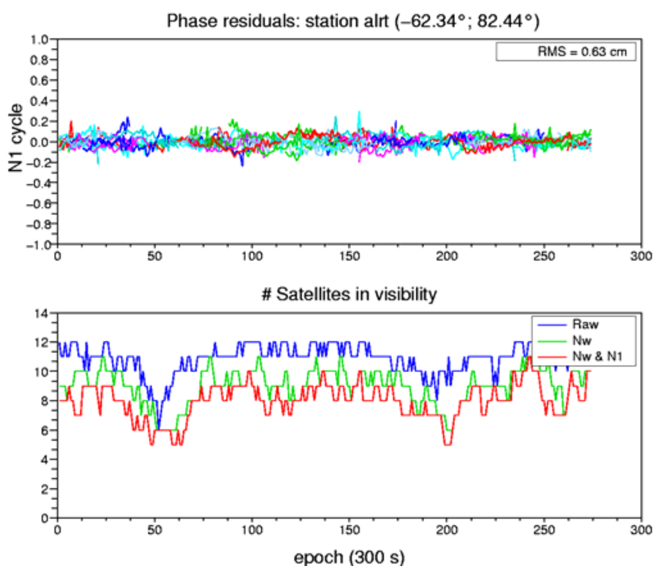

Fig. 4: Station residuals

The upper part of Figure 4 shows an example of ionosphere-free phase residuals as seen by a station (here ALRT) over one day. The RMS is 6.3 mm. There are no visible cycle slips. The lower part of Figure 4 shows the number of satellites in view for all valid data (blue), for measurement with fixed integer widelane ambiguities (green) and for measurements with both integer ambiguities fixed (red). The number of available phase measurements with fixed ambiguities is above 5 at any time, compatible with precision positioning.

4.4. PHASE RESIDUALS PER SATELLITE

Figure 5 shows the ionosphere-free phase residuals for one GPS satellite, in a mirror way to Figure 4 for one station. The number of stations in visibility varies between 5 and 25 along one orbit, with a maximum in the Northern hemisphere. This ensures the continuity of the satellite clock solutions.

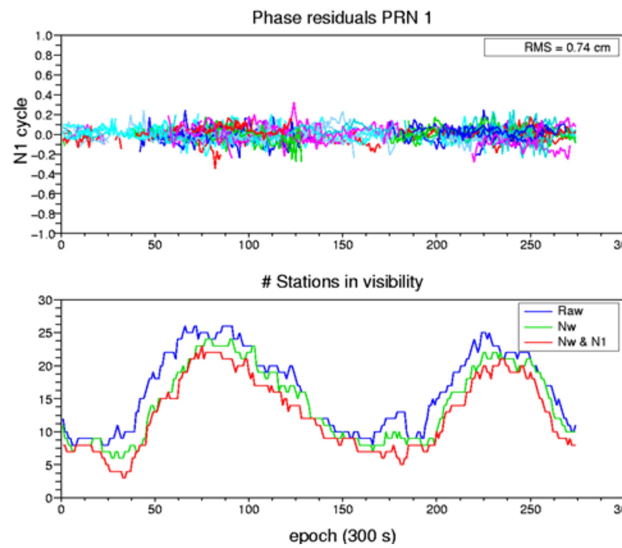

Fig. 5: Satellite residuals

5. STATISTICAL ANALYSIS OF THE ORBIT AND CLOCK SOLUTION

For each of the 120 IGS IGU orbits from July 2007 two sets of real-time corrected orbits and clocks are computed for the GPS constellation. The first one uses zero-difference integer ambiguity fixing (referred to in the following sections as the INT solution). The second one adjusts floating instead of integer ambiguities by simply disabling the integer fixing part of the filter, keeping all other settings unchanged (referred to as FLO in the following sections).

Reference orbits and clocks for filter performance evaluation are the final IGS solutions. The results of the comparison of INT and FLO solutions to this reference are also compared to similar comparisons for IGS IGU and IGR products:

- IGU: IGS IGU solution (extrapolated orbit)
- IGR: IGS IGR solution (rapid orbit produced at D+1)
- FLO: IGU solution corrected by the filter using floating ambiguities
- INT: IGU solution corrected by the filter using integer ambiguities

5.1. LOCAL FRAME ORBIT COMPARISONS

Figure 6 shows the statistics of comparisons of the different solutions to the IGS final orbits in the local orbital frame (radial, along-track, cross-track). For each run, the RMS over the last 12 hours for all satellites is computed.

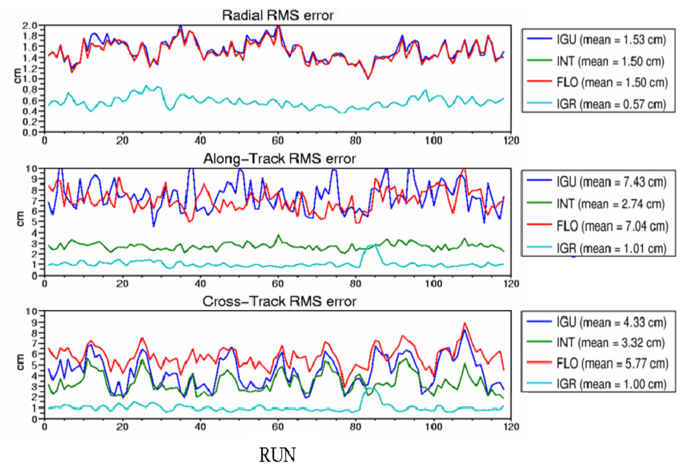

Fig. 6: Local frame orbit comparisons

The radial error is the same for IGU, INT and FLO orbits as this component is not adjusted in the filter. The IGU along-track error is corrected in the INT solution; however, the resulting accuracy is less than that of IGR orbits. The along-track error is not compensated in the FLO solution. A periodic signal is seen in the IGU cross-track comparison; it is partly compensated in the INT solution.

5.2. 3-D ORBIT AND CLOCK COMPARISONS

Figure 7 shows 3-D orbit comparison statistics, as well as clock comparisons. For the clocks, a constant bias per satellite has been adjusted in order to properly estimate the effect in a user solution using floating ambiguities (and also because INT clocks contain an arbitrary offset of an integer multiple of narrow lane wavelengths). A time offset value, for all satellites and each epoch, has also been removed between the solutions, to take into account the case of different reference clocks in the solutions. The IGU predicted clock is not presented because of its poor quality.

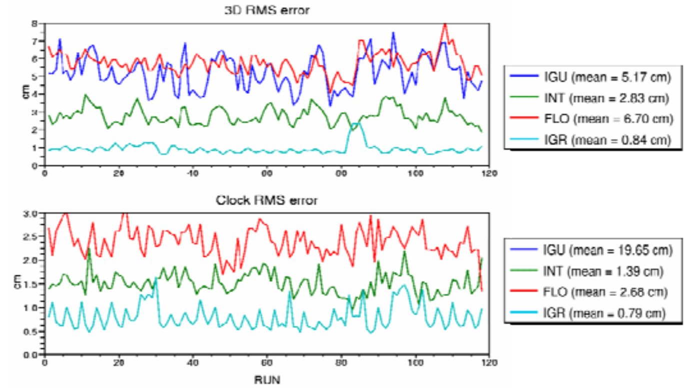

Fig. 7: 3-D and clocks comparison

The INT solution has a good 3-D RMS error (below 3 cm), but it is not nearly as precise as the IGR solution (3-D RMS of 0.8 cm [7]). A similar comparison hierarchy holds for clocks.

5.3. SISRE ANALYSIS

The SISRE (Signal In Space Range Error) is a measure of the quality of the signal seen by a user on the surface of the Earth. This quantity is independent of local phenomenon related to the receiver (measurement noise, multipath).

The contribution of GPS satellites orbit and clock errors to the SISRE, which is a combination of orbit and clock errors projected onto the line-of-sight, is lower than separate errors because of correlations between these quantities. To compute this contribution, it is not necessary to have actual measurements; the line-of-sight can be computed for any arbitrary position on the ground, and two pseudo-range measurements generated, one with the orbit and clock reference solution, the other with the solution to be compared. In addition, a bias per pass is taken into account, in order to simulate the impact of a floating ambiguity positioning solution.

The difference between the value computed with the orbits and clocks under evaluation and the value obtained with the reference orbits and clocks gives the SISRE for this position.

Figure 8 shows the SISRE computed for a virtual position in the South Pacific, where the network contains only few stations. It can be shown that the dispersion of the SISRE with respect to the ground user location is not very significant.

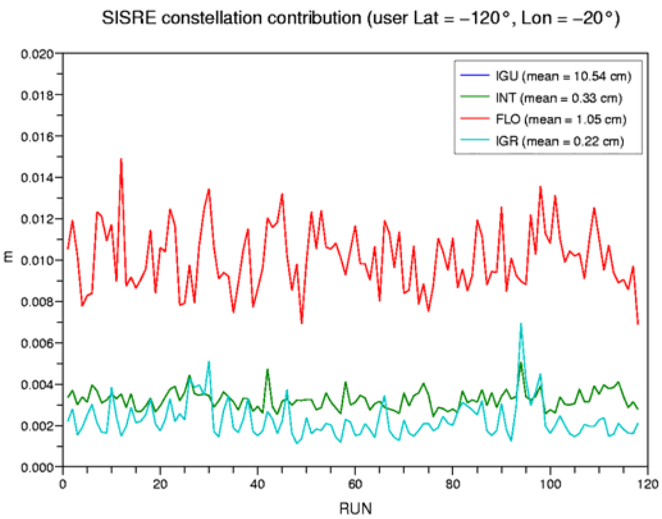

Fig. 8: Constellation SISRE

As expected, the user residuals are much lower than the orbit and clock errors taken separately, with a reduction of an order of magnitude. The SISRE for the integer real-time solution is very close to that of the IGR.

6. POSITIONING PERFORMANCE

Another technique to evaluate the quality of the orbit and clock solution is to process actual measurements from one station and to analyze residuals after precise point positioning. Measurements must come from a station (here “reyk”) which is not part of the network used to compute the orbits and clocks solution.

One day data set of measurements from station “reyk” is processed in a least squares filter to adjust:

- the troposphere vertical delay (1 parameter / 2 hours).
- a stochastic clock at each epoch.
- an ambiguity per pass.

For solutions IGS, IGR and IGU, the ambiguity is estimated as a floating parameter, whereas for the INT solution, the ambiguity is adjusted to an integer multiple of the narrowlane wavelength (widelane integer ambiguities were fixed during pre-processing, consistent with the integer constellation solution).

As the goal of the test is only to inter-compare orbits and clocks of the GPS constellation, it is not necessary to estimate the position of the station. In this test, the station coordinates are set to their ITRF 2005 values.

Figure 9 shows the post-fit residuals obtained with the final (upper plot) and rapid (lower plot) IGS orbits and clocks. Post-fit residuals are of equivalent quality for both input solutions, with residuals around 8 mm.

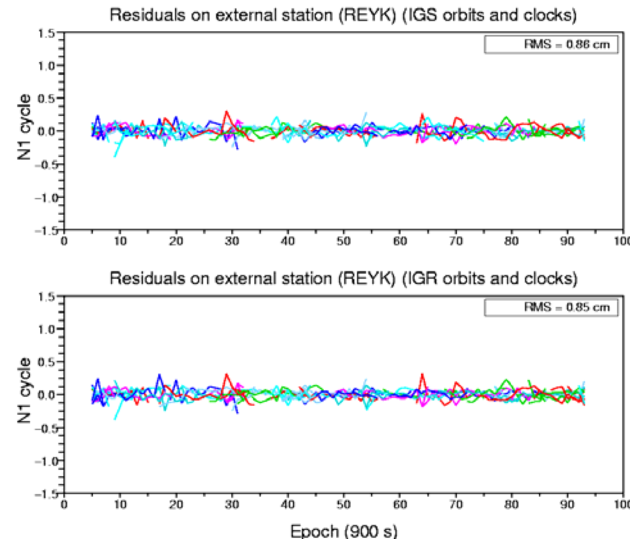

Fig. 9: Post-processed solution residuals

Figure 10 shows the post-fit residuals obtained with the floating ambiguity (FLO, upper plot) and integer ambiguity (INT, lower plot) real-time orbits and clocks. The FLO real-time solution leads to residuals above 2 cm RMS. Meanwhile the INT real-time solution gives residuals close to those of Figure 10, only slightly larger (9 mm RMS). This slight increase is due to the fact that there are fewer degrees of freedom in integer ambiguity solutions compared to floating ambiguity ones. The residual level is well below one N1 cycle (10.7 cm) so the fixing of “reyk” phase ambiguities to their integer values is straightforward.

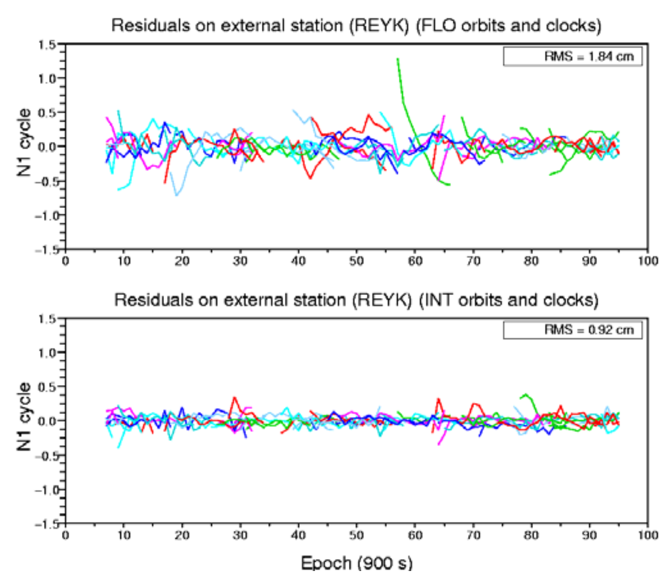

Fig. 10: Real-time solution residuals

In order to estimate the precision of the real-time positioning available to a user of these constellation orbits and clocks, a positioning solution is computed using a Kalman filter which adjusts in real-time the receiver's clock and position, the zenith troposphere delay and an ambiguity per pass, either floating or fixed. Like in the global network solution, in the fixed ambiguity solution, N_1 ambiguities are fixed 'on the fly' once N_w integer ambiguities are known.

The point positioning error is presented on Figure 11. The upper plot shows the user real-time position error along the vertical, East and North directions, computed using the FLO constellation solution and floating ambiguity positioning.

The precision is around 10 cm RMS, similar to what is obtained by standard PPP techniques [10, 11 and 13]. The lower plot in Figure 12 presents the real-time positioning error computed using the INT constellation orbits and clocks and integer ambiguity positioning.

This solution is thus a real-time 'integer' PPP, on a global scale, from end to end. With an error of around 2 cm RMS on each component, this last solution significantly outperforms the standard one (by nearly an order of magnitude).

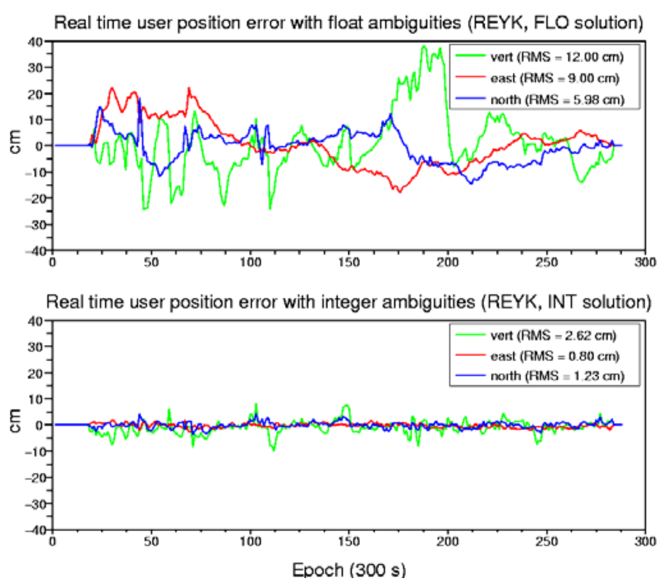

Fig. 11: Real time user position

CONCLUSION

This study demonstrates that the zero-difference integer ambiguity fixing technique [1] can be applied to compute orbits and clocks for the GPS constellation in real-time.

To do so, the Kalman filter initially introduced to estimate the GPS constellation clocks in real-time [3] has been extended to adjust time-correlated GPS satellite orbit corrections.

The filter now simultaneously adjusts corrections to the IGS extrapolated orbits (IGU) and clocks for the GPS constellation, while fixing nearly all integer ambiguities.

The orbit and clock solution output in real-time by the filter supports zero-difference integer ambiguity fixing for user receivers located anywhere in the world.

One month of data was processed to compute real-time orbit and clock solutions and statistically analyze their quality. 3-D orbit errors are around 3 cm RMS, while the orbit and clock contribution to the signal in space range errors (SISRE) is less than 5 mm.

The analysis of residuals for a receiver not included in the network of stations used to produce the constellation orbits and clocks shows that the real-time solution is nearly as precise as the IGS rapid and final products. Residuals are below 1 cm RMS.

In addition, real-time orbits and clocks are compatible with the zero-difference integer ambiguity fixing scheme, leading to real-time worldwide centimeter level positioning.

The filter can be further improved by introducing couplings between radial and along-track orbit errors using Clohesy-Wiltshire equations, leading to dynamically meaningful orbit corrections.

It is also possible to go one step further to completely estimate the orbit without the help of an initial solution. Such a strategy would have the advantage of autonomously providing precise orbits and clocks for the GPS constellation in real-time.

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