

An Open-source PPP Client Implementation for the CNES PPP-WIZARD Demonstrator

D. Laurichesse, *Centre National d'Etudes Spatiales, Toulouse, France*
A. Privat, *CS-SI, Toulouse, France*

BIOGRAPHY

Denis Laurichesse is a member of the navigation system department at CNES. He has been in charge of the DIOGENE GPS orbital navigation filter, and is now involved in navigation algorithms for GNSS systems. He is currently in charge of the CNES IGS real time analysis center. He was the co-recipient of the 2009 ION Burka award for his work on phase ambiguity resolution.

Aude Privat is an engineer of the ground system engineering service at CS-SI. She is currently in charge of the navigation projects in her service. She is involved in the development of navigation algorithms for GNSS systems with CNES.

ABSTRACT

In the framework of the IGS real-time service (RTS), CNES is now proposing for research purposes a new open source implementation of its PPP client. This PPP client corresponds to the user part of the PPP-WIZARD (With Integer And Zero-Difference Ambiguity Resolution) demonstrator.

The paper presents this new product in details. Its architecture is designed to be as flexible as possible. Core algorithms are implemented in a library, compatible with other open sources implementations like BNC and RTKLIB. The architecture allows easy replay mode processing as well as parallel processing of a network of receivers. The C++ language was chosen for the implementation to allow high-level programming while maintaining a good level of portability, even on embedded devices with low CPU power. Special attention has been paid to ensure rapid execution on all platforms.

The paper also details the functional model. This model exploits the concept of undifferenced and uncombined measurements. It is compatible with the most the latest state of the art techniques, such as ambiguity resolution, fast reconvergence, gap bridging, seamless regional

augmentation, GNSS centric. It fully exploits the possibilities of the State Space Representation promoted by the RTCM standard.

The estimation process is based on an extended Kalman filter. The parameters of the filter, i.e. the state vector, which reflect the functional model, are described. The filter process is split in two phases, propagation and estimation, that are also described, with an emphasis on the state estimation process based on an accurate measurement model. A typical tuning of the filter is proposed.

Ambiguity resolution is an important aspect of PPP. Our model is by construction compatible with zero-difference ambiguity resolution. Since the formulation is uncombined, it is not only compatible with dual-frequency systems, but also paves the way for triple-frequency ambiguity resolution and the faster convergence time it allows reaching.

Another important aspect of a PPP client is its robustness: to do so, several advanced algorithms, such as standard RAIM, advanced RAIM and gap bridging are implemented. These algorithms are detailed and typical improvement results are shown.

Doppler smoothing is also implemented in the tool. This feature is particularly useful for single frequency applications, where the positioning solution is highly dependent on the quality of pseudo-range measurements. It is shown that adding Doppler data improves the quality of the solution in such a configuration.

This PPP implementation is compatible with regional augmentation, thanks to the use of a specific atmospheric interface. This interface includes zenithal tropospheric delay and slant ionosphere delay. It is shown that this information, obtained for example from a dense network, allows reaching centimeter accuracy in a few minutes

The constellations currently supported are those broadcast by the IGS RTS service i.e. GPS and Glonass; there is an on-going work to add Galileo and Beidou.

Finally, results obtained with real data using various configurations are presented. These configurations include comparisons between single and dual frequency processing, standard or precise point positioning, and using global, regional or local atmospheric augmentation. The results show the typical performance that can be obtained using PPP technology in terms of convergence time and accuracy after convergence.

1. INTRODUCTION

In its role of IGS Real-Time Service analysis center [6], CNES provides accurate orbits and clocks for GPS and Glonass (Product prefix CLK9x) on a routine basis. The particularity of the CNES solution is that it implements a technique called ‘undifferenced ambiguity resolution’ [7]. This technique improves the intrinsic quality of clocks for the IGS Real Time Service. The use of these clocks allows ambiguity resolution on any isolated “user” receiver enabling it to achieve 1 cm horizontal precise point positioning accuracy. In order to evaluate the quality of these real time products, CNES is now making available for research purposes a new open source implementation of its PPP client. The main purpose of this software is to provide a tool compatible with the CLK9x, to fully assess the quality of these products.

2. FEATURES

The software implements the following features:

Table 1. PPP module features

Functional model	Undifferenced and uncombined
Measurements	Code, Phase, Doppler
Constellations	GPS, Glonass for now ongoing Galileo and Beidou
Frequency	Mono, Dual, Triple
Ambiguity resolution	Dual or Triple (GPS) frequency Bootstrap method Cascading: Extra-widelane, widelane, N1
Ionosphere source	SBAS via RTKLIB
Orbits/Clocks corrections	RTCM or SBAS via RTKLIB
RAIM	improved
Gap-bridging	YES
Multi-receivers processing	YES
Portable architecture	YES (C++, STL)

Compatibility	BNC, RTKLIB
Doppler smoothing	YES
Code biases source	RTCM, Tgd(BRDC), DCB via RTKLIB
Phase biases source	RTCM
Regional augmentation	YES (via RTROVER interface)
Replay mode	YES
Constellation centric Frequency centric	YES
Clock synchronization	YES
Platforms	Linux, windows

The software is available in the format of a C++ library. C++ language has been chosen for the implementation to allow high-level programming while maintaining a good level of portability, even on embedded devices with low CPU power. The software is self-sufficient, i.e. it does not use other libraries. Attention has been paid to ensuring compatibility with other open source software like BNC or RTKLIB. Orbit and clocks corrections come mainly from RTCM (real-time streams) but the software is capable of handling SBAS corrections as well. Ambiguity resolution is an important feature of the software.

3. ARCHITECTURE

The overall architecture of the software is represented in the following figure:

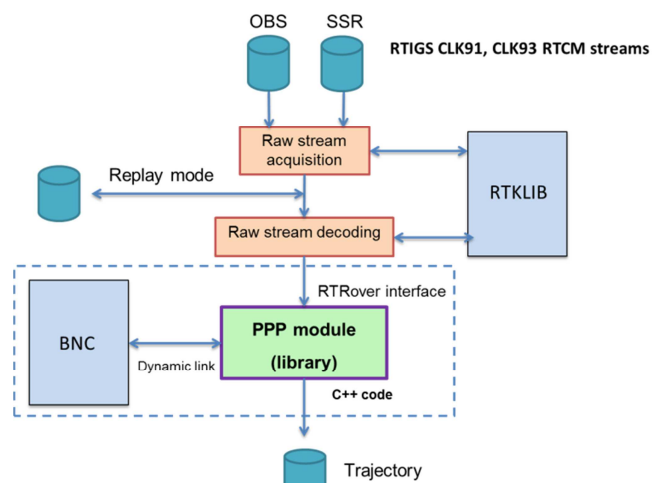


Fig. 1: Overall PPP architecture

In order to be compatible with BNC [2], the core PPP module is implemented as a library and respects the RTRover interface: this allows linking directly the module with BNC. In addition, this interface can also be fed with RTCM streams of observables and corrections. In this case, the decoding of the streams is performed thanks to the RTKLIB library [11]. A separate module using also RTKLIB is in charge of the streams acquisition. In this way, the streams can be directed to the PPP module or stored for further replay.

The core PPP module is represented on the following figure:

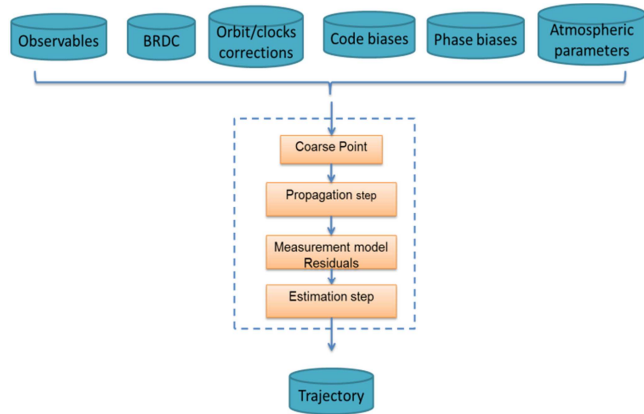


Fig. 2: PPP module task flow

The RTRover interface provides to the module the different inputs:

- Observables
- Broadcasts
- Corrections
- Biases
- Atmospheric data

Then the computation step of the interface is invoked. This step performs the following tasks:

- Computation of an initial coarse point
- Propagation of the state vector
- Computation of the residuals thanks to the measurement model
- Estimation of the state vector

Finally, the computation step outputs the solution. The solution is composed of the receiver's position as well as the troposphere delay and the associated covariances.

4. FUNCTIONAL MODEL (GPS CASE)

In this section, we review the formulation of the observation equations. We will use the following constants in the equations [5, 7]:

f_1 , f_2 and f_5 are the three frequencies transmitted by GPS satellites and c is the vacuum speed of light. For the GPS L1, L2 and L5 bands, $f_1 = 154f_0$, $f_2 = 120f_0$ and $f_5 = 115f_0$, where $f_0 = 10.23$ MHz. The pseudorange (or code) measurements, P_1 , P_2 and C_5 , are expressed in meters, while phase measurements, L_1 , L_2 and L_5 , are expressed in cycles. In what follows, the word "clock" stands for a time offset between a receiver or satellite clock and GPS System Time as determined from either code or phase measurements on different frequencies or some combinations of them.

The notations are:

$$\gamma_2 = \frac{f_1^2}{f_2^2}, \quad \gamma_5 = \frac{f_1^2}{f_5^2}, \quad \lambda_1 = \frac{c}{f_1}, \quad \lambda_2 = \frac{c}{f_2}, \quad \lambda_5 = \frac{c}{f_5}$$

$$P_1 + \Delta b_{p_1} = D_1 + e + \Delta h$$

$$P_2 + \Delta b_{p_2} = D_2 + \gamma_2 e + \Delta h$$

$$C_5 + \Delta b_{c_5} = D_5 + \gamma_5 e + \Delta h$$

$$\lambda_1 (L_1 + \Delta b_{l_1}) = D_1 + \lambda_1 W - e + \Delta h - \lambda_1 N'_1$$

$$\lambda_2 (L_2 + \Delta b_{l_2}) = D_2 + \lambda_2 W - \gamma_2 e + \Delta h - \lambda_2 (N'_w + N'_1)$$

$$\lambda_5 (L_5 + \Delta b_{l_5}) = D_5 + \lambda_5 W - \gamma_5 e + \Delta h - \lambda_5 (N'_E + N'_w + N'_1)$$

$$N'_1 = N_1 + \delta N_1$$

$$N'_w = N_w + \delta N_w$$

$$N'_E = N_E + \delta N_E$$

where:

D_1 , D_2 and D_5 are the geometrical propagation distances between the emitter and receiver phase centers at f_1 , f_2 and f_5 including troposphere elongation, relativistic effects, etc.

W is the contribution of the wind-up effect (in cycles).

e is the ionosphere elongation in meters at f_1 . This elongation varies with the inverse of the square of the frequency and with opposite signs between phase and code.

$\Delta h = h_i - h^j$ is the difference between receiver i and emitter j clocks. This clock is arbitrary and can follow any definition, if the code and phase biases are consistent [5], but in the IGS convention these clocks are referenced to the ionosphere-free P1-P2 combination. N'_1 , N'_w and N'_E are the three carrier phase ambiguities, respectively N_1 Widelane and Extra-widelane. By definition, these ambiguities are integers.

The δN_1 , δN_w , δN_E quantities are also integer values, introduced for technical purposes (gap-bridging capability).

In routine mode, these quantities are set to 0.

$\Delta b_p = b_{p_i} - b_{p_j}$ is the difference between receiver i and emitter j code biases, as defined in the RTCM standard [9].

$\Delta b_L = b_{L_i} - b_{L_j}$ is the difference between receiver i and emitter j phase biases, as defined in the RTCM standard [10].

In general, the Δ operator represents a difference between a receiver and emitter quantity. In this model, there is one receiver quantity per constellation.

The main difference between this formulation and the classical one is that it is not combined. The ionosphere elongation is explicit in the equations and allows the use of external ionosphere information enabling fast convergence and gap bridging. Integer ambiguity appears explicitly in the model in an undifferenced formulation.

5. COARSE POINT

The initial point (which is used as an initial condition for the linearization of the filter's equations) is computed at each epoch, using the code measurements and an iterative least squares estimation. The estimated accuracy of such a point is about 100 m, which is inside the linear domain of the filter.

6. KALMAN FILTER

The general formulation of the Kalman filter is of the $P = UDU'$ form, with U an upper triangular matrix and D a squared diagonal matrix. The update of the solution matrix follows the Bierman algorithm [1], while the propagation of the solution uses the Thornton algorithm [12].

The parameterization of the filter follows the functional model. The satellite part of the functional model quantities (positions, clocks, biases) are known and thus are not identified. Only the receiver parameters are estimated in the state vector.

For practical reasons, the receiver clock is directly estimated in the biases ($h_i = 0$).

The following table represents the state vector and tuning of the filter (for the two constellations GPS and Glonass but it can be easily generalized to other constellations):

Table 2. Filter parameterization

Parameter	Unit	Quantity	Typical initial covariance	Typical model noise (1 s)
Position	m	3	50 m	10 m
Clock P1 GPS	m	1	inf	inf
Bias P2 GPS	m	1	0	1 mm
Bias C5 GPS	m	1	0	1 mm
Bias L1 GPS	m	1	0	1 mm
Bias L2 GPS	m	1	0	1 mm
Bias L5 GPS	m	1	0	1 mm
Clock (P1) GLO	m	1	inf	inf
Bias P2 GLO	m	1	0	1 mm
Bias L1 GLO	m	1	0	1 mm
Bias L2 GLO	m	1	0	1 mm
Zenithal tropospheric delay	m	1	0.5 m	0.005 mm
Ionospheric delay (slant)	m	1 per satellite (70)	10 m	2 mm
Phase ambiguity Ne	Cycle	1 per satellite (70)	inf	0
Phase ambiguity Nw	Cycle	1 per satellite (70)	Inf	0
Phase ambiguity N1	Cycle	1 per satellite (70)	inf	0
Phase ambiguity dNe	Cycle	1 per satellite (70)	0 routine inf if gap- bridging	0
Phase ambiguity dNw	Cycle	1 per satellite (70)	0 routine inf if gap- bridging	0
Phase ambiguity dN1	Cycle	1 per satellite (70)	0 routine inf if gap- bridging	0

7. PROPAGATION STEP

The propagation step of the filter is relatively simple because the transition matrix of the parameters equals the identity. Thus, this step consists only in the addition of a diagonal model noise on each parameter.

8. CORRECTION STEP

The correction step of the Kalman filter involves a modeled measurement set at the current epoch. The targeted accuracy for the measurement models has been set to 5 mm. To do so, the following terms and models are computed:

Table 3. Measurement model

Model term	Amplitude (m)	Comment
Propagation distance	107	Incl. Sagnac effect
Solid earth tide	0.10	Routine dehanttideinel.f
Phase Wind-Up	0.10	Wu & al. [13]
Troposphere	0.10	Zenithal delay Mapping function: Saastamoinen
Attitude law	0.10	Nominal
Satellites and stations PCV and PCO	0.01	igs08_www.atx
Relativistic effects	0.005	2nd order correction on clocks gravitational time delay

9. AMBIGUITY RESOLUTION

The integer nature of ambiguities is preserved thanks to the use of the appropriate phase biases RTCM message. The identification of the integer ambiguities is performed with the bootstrap method [3]: after an initial a priori fix, the remaining ambiguities are fixed iteratively. This technique allows partial ambiguity fixing. To facilitate the estimation process, ambiguities are fixed in a cascading scheme: first the extra-widelane N_e , then the widelane N_w and then N_1 . A new set of integer ambiguity is estimated at each epoch to avoid staying on a wrong ambiguity solution from one epoch to another.

10. CONVERGENCE RESULTS

This section presents typical results of the PPP module during the convergence phase. For these tests, a triple-frequency, dual constellation receiver is used. RTCM SSR

corrections are sent to the PPP module, as well as regional ionosphere information pulled from the EGNOS system.

The typical convergence profile (horizontal accuracy) for the single frequency configuration is represented figure 3 for four modes /runs :

- GPS only (dark blue)
- GPS + Glonass (green)
- GPS + Glonass and clocks corrections (red)
- GPS + Glonass, clocks correction and EGNOS ionosphere (cyan blue)

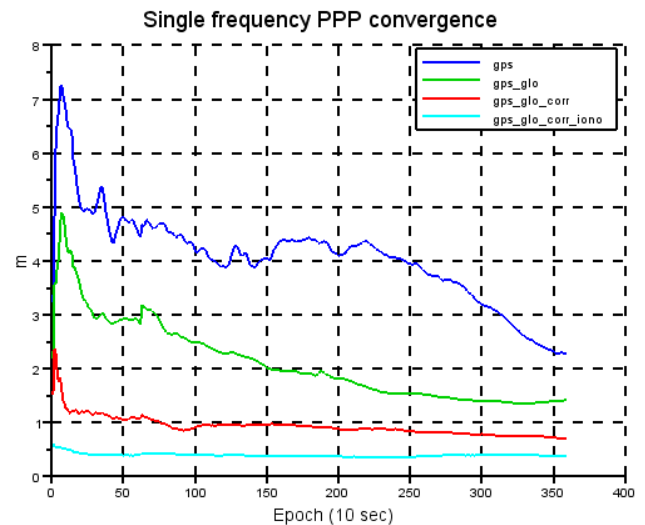


Fig. 3: Typical single frequency convergence

These runs show that the use of the accurate clocks allows a faster convergence. The most interesting combination is given by the use of both the accurate clocks and regional ionosphere. In this case, 50 cm centimeter accuracy is obtained instantaneously. The following table summarizes these results:

Table 4. Convergence (single frequency)

Configuration	Horizontal accuracy (RMS, cm)	Vertical accuracy (RMS, cm)	Convergence time (horizontal < 1 m)
gps	59.47	78.75	n.a.
gps_glo	69.88	83.19	2 h
gps_glo_corr	23.31	33.62	30 min
gps_glo_corr_iono	24.80	27.99	1 min

Typical convergence profile for the dual-frequency configuration is represented in figure 4 for three modes / runs:

- GPS only + clocks correction (dark blue)
- GPS + Glonass + clocks correction (green)
- GPS + Glonass + clocks correction and EGNOS ionosphere (red)

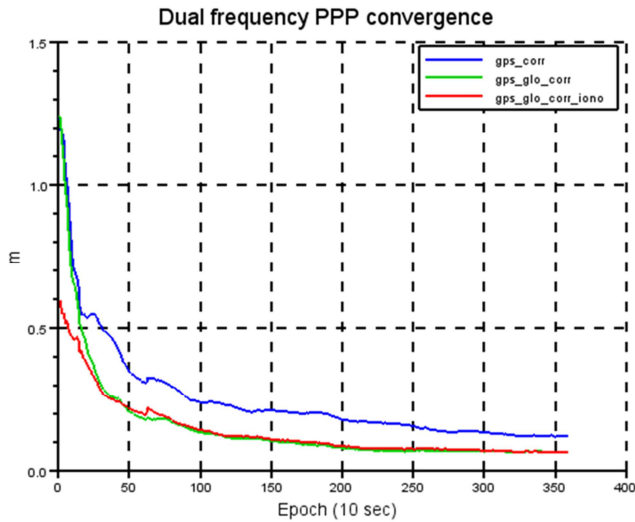


Fig. 4: Typical dual-frequency convergence

An important improvement of the convergence time is noted when Glonass is added. It is expected that convergence time will be reduced even further when other constellations will be available. It is also noted that, thanks to the uncombined formulation, there is a slight improvement when the ionosphere information is used, even in the dual frequency context.

The following table summarizes these results:

Table 5. Convergence (dual-frequency)

Configuration	Horizontal accuracy (RMS, cm)	Vertical accuracy (RMS, cm)	Convergence time (horizontal < 10 cm)
gps_corr	2.87	4.42	45 min
gps_glo_corr	2.64	3.66	20 min
gps_glo_corr_iono	2.65	3.68	20 min

Another study carried out in the triple-frequency context [5], shows that the use of the third GPS frequency greatly improves the convergence time. Figure 5 shows typical convergence profiles in a configuration where 4 GPS IIF were simultaneously in view (each color represent a different

convergence run). The convergence time is reduced to a few minutes. No conditions for ambiguity resolution are set, which explains the noisy patterns during the convergence.

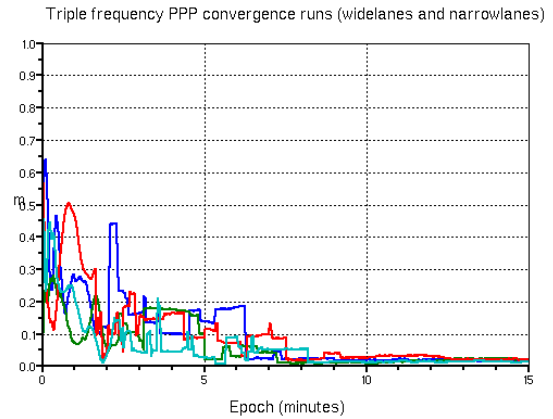


Fig. 5: Triple-frequency convergence (4 blocks IIF in view)

Regional augmentation also allows the reduction of convergence time. The PPP module has an interface for local a-priori troposphere and ionosphere information. It is therefore possible to test several configurations using this information. Four runs were made:

- No information
- Troposphere only
- Ionosphere only
- Troposphere + ionosphere

It is assumed here that this atmosphere information is perfectly known. To do so, we conduct a first PPP run (with the receiver fixed to its reference value), get the atmosphere information, and then carry out a second run with this information representing the a-priori values.

Figure 6 shows the convergence profile for the different runs.

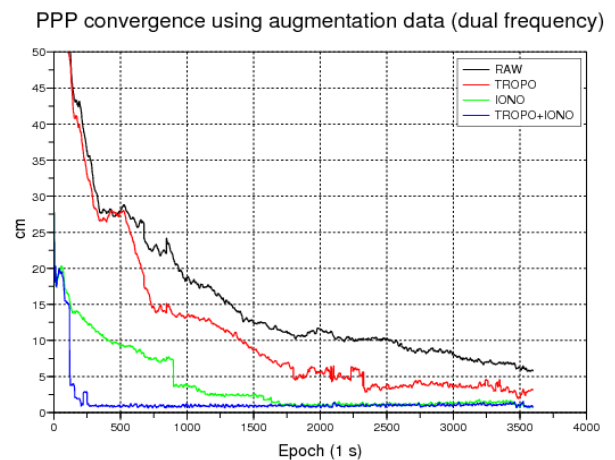


Fig. 6: Typical convergence using regional augmentation

It is clear that the use of both the troposphere and ionosphere information allows a very rapid convergence (between 1 and 2 minutes).

11. ROUTINE RESULTS

After convergence, when GPS ambiguities are fixed, centimeter accuracy is routinely achieved, as shown in the figure 7, which is a snapshot of the PPP-Wizard web site [8].

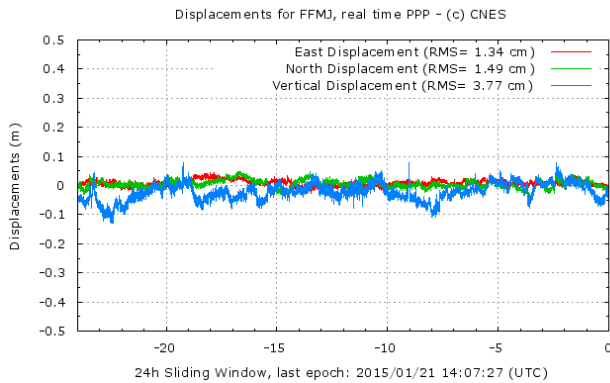


Fig. 7: Routine dual-frequency accuracy (AR)

12. RAIM

The RAIM (Receiver Autonomous Integrity Monitoring) is a generic mechanism to detect and possibly exclude false measurements at the receiver level. It is in general based on measurements redundancy. In this implementation, two FDE (Fault Detection and Exclusion) mechanisms are implemented, on both code and phase measurements:

Simple RAIM: Generally speaking, a false measurement is detected when the measurement residual is above a certain threshold. However, as described in figure 8, an erroneous measurement can contaminate the others residuals and may lead to wrong exclusions. This is why an iterative elimination is performed: after each elimination, a new solution (and new residuals) is computed.

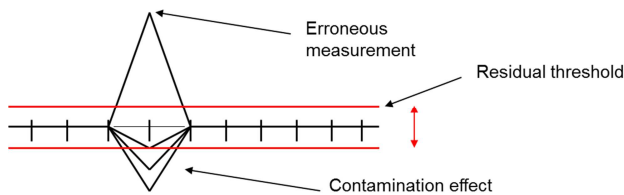


Fig. 8: Simple RAIM detection

Figure 9 shows a typical positioning result using the simple RAIM algorithm, with a station in rough conditions.

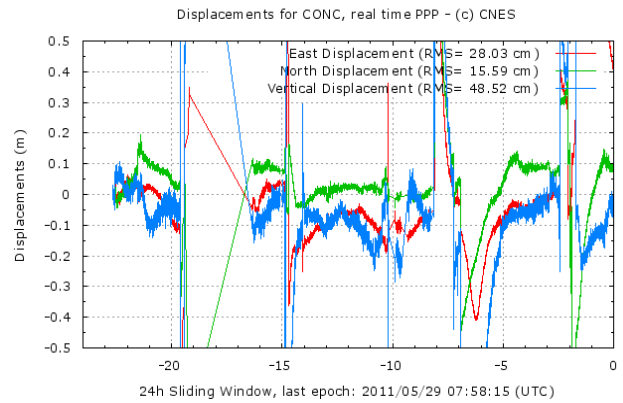


Fig. 9: Outlier detection (simple RAIM)

Advanced RAIM: The idea of this algorithm is to remark that it is not necessarily the erroneous measurement that gives the worse residual i.e. the residual of the erroneous measurement can be below the threshold (figure 10).

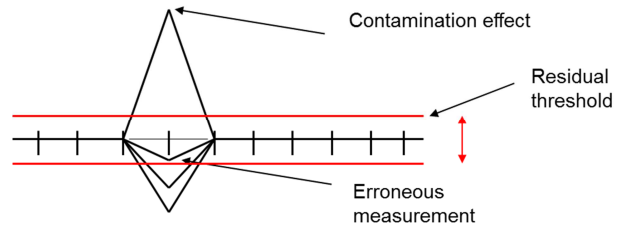


Fig. 10: Advanced RAIM detection

The principle of the algorithm consists in testing all the combinations of measurements to remove (one by one, then two by two, etc.) until a combination where all the remaining residuals are below the threshold is found. Figure 11 shows the results of the advanced RAIM algorithm. The reduction of outliers is clearly visible.

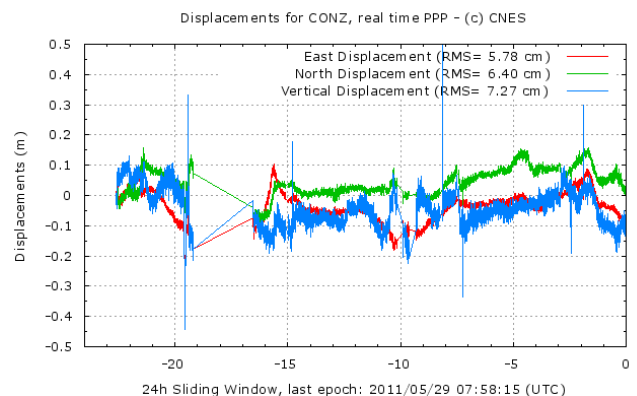


Fig. 11: Outlier detection (advanced RAIM)

13. GAP-BRIDGING

Gap-bridging is a technique that allows recovering from measurement gaps or from an initialization of the receiver [4]. The set of ambiguities may be completely different before and after the gap. The typical duration of a gap is less than one minute. This event generally occurs when the receiver passes through a tunnel or under a bridge.

During the gap, the receiver is capable of maintaining some of its internal state (typically the troposphere and the ionosphere), to ensure a fast reconvergence. In addition, if one assumes that the ambiguity jumps are of integer values, it is possible to estimate these jumps. It is worth noting, however, that this functionality does not need the individual ambiguities to be estimated as integer values. In particular, this is the reason why the technique also works for the Glonass constellation.

Jump estimation is possible thanks to the estimation of δN parameters of the filter and a bootstrap resolution.

The overall mechanism is described in the following figure:

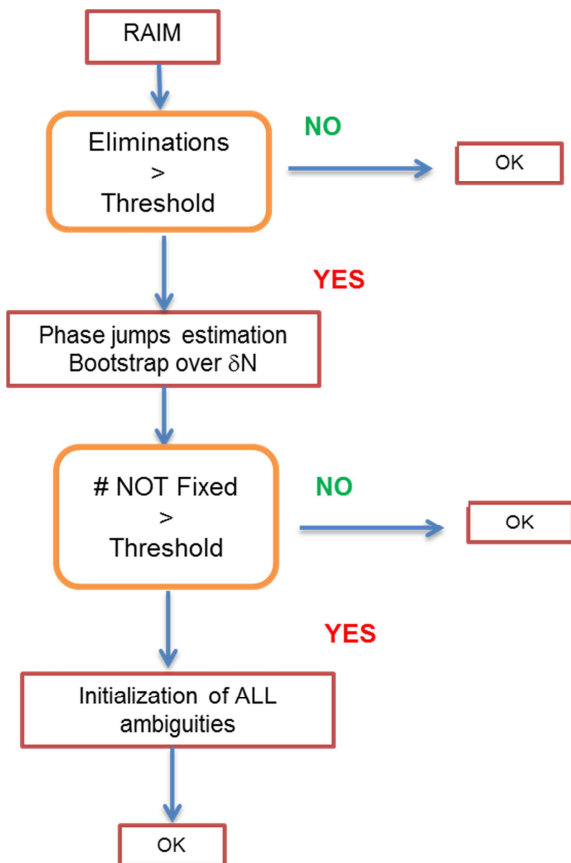


Fig. 12: Gap-Bridging diagram

The following figures (13 and 14) represent a study where artificial jumps are introduced every hour in a PPP run (with a zero-length gap), which is clearly visible in the first figure. In the second figure, both atmospheric parameters and jumps

are estimated. It clearly shows the improvements of the gap-bridging algorithm.

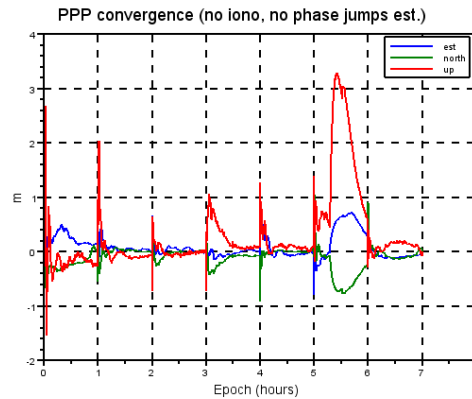


Fig. 13: 1-hour artificial jumps without gap-bridging

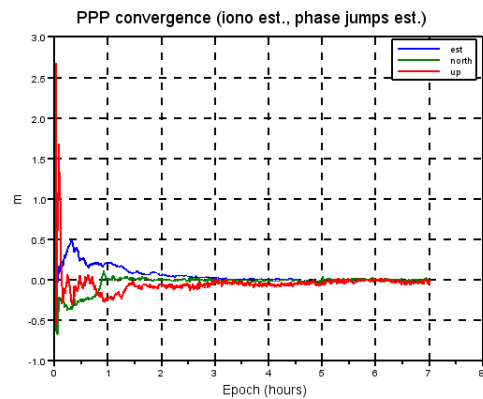


Fig. 14: 1-hour artificial jumps with gap-bridging

The duration of the gap is also an important factor of success. Figure 15 represents the success rate of the algorithm (percentage of corrected estimated jumps) w.r.t. the gap duration. It shows that, in order to achieve a good success rate, the gap should not exceed 2 minutes.

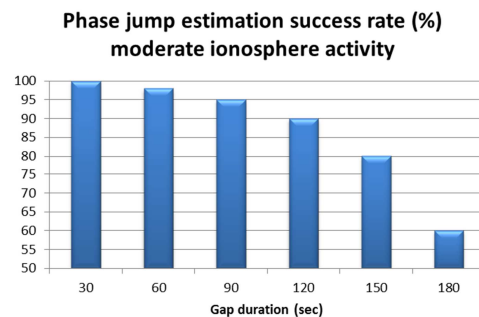


Fig. 15: Gap-bridging success rate

14. DOPPLER SMOOTHING

Code measurement is in general noisy. This noise is particularly visible in single-frequency solution, where it is not possible to perform an ionosphere-free solution with the phase measurements only. However, it is possible to smooth these code measurements thanks to the Doppler measurements. The aim of this smoothing is to reduce the noise and not to reduce the bias. This smoothing is performed independently on each frequency with a filter of order one:

$$C_{St} = (1 - \alpha)C_t + \alpha \left(C_{S(t-dt)} + dt \frac{D_{t-dt} + D_t}{2} \right)$$

with :

C_{St} Smoothed code measurement (at t)

C_t Raw code measurement (at t)

$C_{S(t-dt)}$ Smoothed code measurement (at t-dt)

D_t Doppler measurement (at t)

D_{t-dt} Doppler measurement (at t-dt)

α Smoothing coefficient (0 not smoothing, 0.95 smoothing)

The following figures (16 and 17) present the result of such a smoothing in a single-frequency context (respectively not smoothed and smoothed):

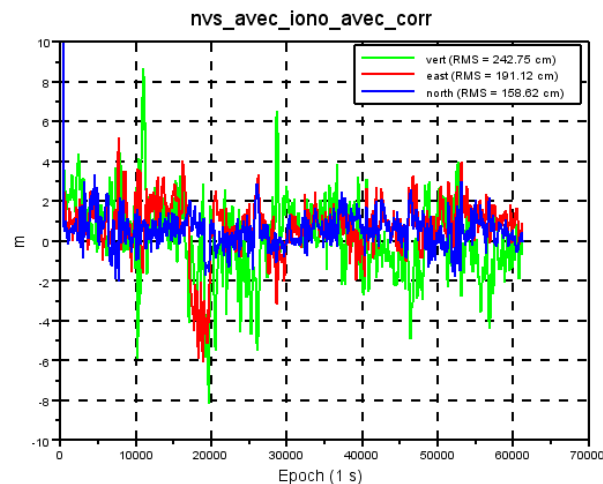


Fig. 16: RAW single frequency PPP

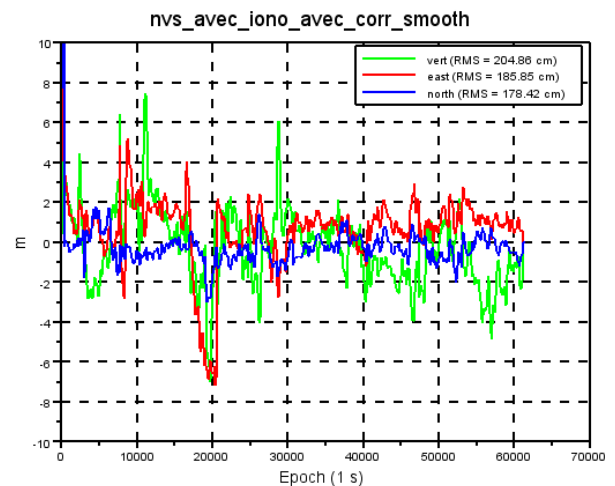


Fig. 17: Doppler-smoothed single frequency PPP

15. OTHER FEATURES

- Constellation independent, frequency independent
For symmetry reasons, there is no preference on a specific constellation or a specific frequency. The user can for example select to perform a PPP solution on GLONASS only. In addition, this allows the computation of a solution even if less than 4 satellites are in view for each constellation (for example 3 GPS and 2 Glonass).

- Multi-receivers
The multi-receivers feature allows to process a whole set of receivers at a time, particularly useful for hazard monitoring.

- Clock correction synchronization
Accurate clock corrections available at the RTS project follow the RTCM convention and are time-tagged. In order to perform a precise positioning, these clocks need to be recent. A new mechanism is implemented to ensure that the delay between the clock corrections and the time of measurements does not exceed a certain threshold (typically 10-20 seconds). This delay is modifiable in the settings. It consists in buffering the measurements until the clocks correction are refreshed. This functionality allows reducing the short term noise of the solution.

16. CONCLUSION

The proposed PPP implementation is available in an open-source format under license for non-commercial users. As of 2015-04-28, the current version of the software is 1.2. Interested users can follow the instructions to download the package at the address of the project:

<http://www.ppp-wizard.net/package.html>

The content of the package is the following:

- The PPP library code
- The modified rtklib code
- The modified BNC code as well as a static executable for Windows
- Code for stream acquisition and stand-alone executable
- Documentation and examples

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